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Modelling of Direct Current (DC) Motor for Performance Improvement using Model Parameter Estimation

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ABSTRACT
A key concern with the optimal use of direct current (DC) motor is model updates. In this article, DC motor models from probability density function (PDF) data have been obtained using the system identification method in MATLAB. At first, noise-free input and output data were used to obtain the parameters of the AutoRegressive with eXogenous input (ARX) model structure. Thereafter, the DC motor subjected to random noise had its input and output data used to obtain the ARX model parameters. However, the resulting model from the noisy data differed from the noise-free data model. To minimize this difference, 200 input/output data set for the noisy DC motor were generated and used to obtain the model parameter for each data set. The mean of the 200 data set was obtained, and the resulting model approached the noise-free model as much as possible. The root-mean-squared error of prediction stands at $3.9426E^{-17}$, $3.35E^{-2}$, and $3.15E^{-2}$ for noise-free, noisy, and mean of noisy DC motor model, respectively, showing accuracy of result for noise-free system. The article provides a way by which the model of the DC motor can be updated when it is not convenient or possible to measure the DC motor parameters when used for motion actuation. The parameters of the selected model structure are therefore estimated from the DC motor input/output data instead of the DC motor parameters itself.
Keywords: DC Motor, PDF Data, System Identification, modelling,ARX Model

I.0 INTRODUCTION

System modeling is a key element of most design and simulation applications, which enables designers to examine whether design specifications are met (Sinha et al., 2001). From control systems (Maheriya and Parikh, 2016) to mechanical (Guessasma et al., 2011), chemical (Simorgh et al., 2020), electrical (Hosseini et al., 2021), mining (Liu et al., 2023), and economy (Lanz and Rutherford, 2016), among others, models play a role in explaining cause-and-effect relationships, operator training, and simulation, among others. Models are obtained from the first principle, providing the white box models, or from system identification, providing the black box models (Ljung 2010). In white box models, the fundamental principles of the system to be modeled are known, and the system is represented by sets of equations based on scientific, economic, and social laws as appropriate. In blackbox models, however, the knowledge of the physical system to be modeled is not known (Keesman, 2011).

A DC motor is the major actuator for motion control in most industrial and robotic applications (Thierry and Audouy, 2021). The modeling of the system is usually done from first principles or white box modeling, as the physics of DC motors is established (Adukwu and Oku, 2018). However, due to usage, noise, and environmental factors, some of the DC motor parameters, like friction coefficient and electrical constant, among others, degrade in value; hence, the model obtained from the first principle becomes suboptimal when used for control applications. One way to reduce this effect is to regularly measure these parameters and update the DC motor model periodically or at intervals, as decided by Fazdi and Hsueh (2023). This could, however, impact the smooth running of the system and, in some cases, be practically impossible as sensors for these measurements have to be deployed in the system.

Another approach is to replace the first principle modeling with system identification, or black box modeling. In this approach, the input and output data of the DC motor are taken at a given interval, and then a model structure is assumed for the DC motor. The parameters of these models are obtained and substituted into the model structure. The obtained model is validated with separate data from the modeling data to obtain the system model. While this requires lots of memory space (to store measurement data) and computational efforts (to obtain the model parameters), it is more realistic than the measurement of DC motor parameters because this can be done both online and offline without disturbing the operation of the system.

In this paper, a DC motor model from black box modeling is obtained. The DC motor is a simple, second-order linear time-invariant system without load torque disturbance. ARX (autoregressive exogenous) model structure is selected (Saifizi et al., 2013). By generating 32 data points, the parameter for the ARX model of the system is obtained and substituted into the model structure. This is done for both the noise-free and noisy DC motors. The key contribution of this work is the provision of an alternative approach to updating the DC motor model in application areas where the model changes quickly due to changes in DC motor parameters and there are limitations in deploying sensors to update these parameters.

The rest of this article is as follows: Section II is the theoretical analysis where the DC motor is explained and the first principle model is presented. Theoretical knowledge of the ARX model structure is presented here. Section III presents the materials and methods by which the ARX models and estimations of both the noise-free DC motor and the noisy DC motor are obtained. Section IV discusses the model of the DC motor obtained from the parameters in Section III. Section V presents the conclusion and recommendation.

1.1 THEORETICAL ANALYSIS

A. The DC motor

The DC motor is presented in Figure 1. The input is the supplied voltage V, and the output is the angular vel ocity w. Based on Adukwu and Oku (2018) and ignoring the load torque, the transfer function of the DC motor in continuous time is given as:

$$G(s) = \frac{K_2}{JLs^2 + (JR + bL)s + (bR + K_1K_2)} \tag{1}$$

Where the parameters are:
J=inertia constant, b=damping coefficient, R=armature resistance, L=loop inductance, K_1 =electrical constant and K_2 =torque constant.

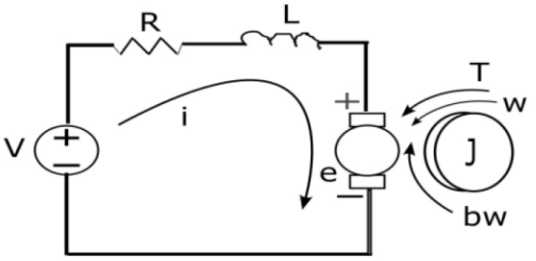

Figure 1. Simplified DC motor

Using the parameters provided in Adukwu and Oku (2018), this transfer function is given as:

$$G_c = \frac{20}{s^2 + 20.5s + 50} \tag{2}$$

The discrete time equivalent of (2) using a sampling time of h=0.1 is given as:

$$G_d = \frac{0.0541z + 0.0276}{z^2 - 0.9244z + 0.1287} \tag{3}$$

In the normal form, the transfer function in (3) can be written as:

$$y_k - 0.9244u_{k-1} + 0.1287y_{k-2} = 0.0541u_{k-1} + 0.0276u_{k-2} \tag{4}$$

Where G_c is the continuous time transfer function of the DC motor, G_d is the equivalent discrete time transfer function while s and z show the system dynamics in continuous and discrete domains respectively, y_k where i=1,2….n is the delayed output by 1,2…n. The same is true for the input delay U_{k-Ii}.

B. ARX Model of the DC Motor

Following from (4), the model of an nth-order discrete time system in normal form is therefore:

$$y_k + a_1y_{k-1} + a_2y_{k-2} + \cdots + a_ny_{k-n} = b_1u_{k-1} + b_2u_{k-2} + \cdots b_nu_{k-n} \tag{5}$$

Introducing the shift-operator $z^ix = x_{k+i}$, then (5) becomes:

$$y_k + a_1z^{-1}y_k + a_2z^{-2}y_k + \cdots + a_nz^{-n}y_k = b_1z^{-1}u_k + b_2z^{-2}u_k + \cdots b_nz^{-n}u_k \tag{6}$$

This written in compact form gives:

$$C_ay_k = C_bu_k \tag{7}$$

Where;

$C_a = 1 + a_1z^{-1} + a_2z^{-2} + \ldots + a_nz^{-n}, C_b = b_1z^{-1} + b_2z^{-2} + \ldots + b_nz^{-n}$ (8)

The model in prediction form is given from (6) as:

$$y_k = -a_1z^{-1}y_k - a_2z^{-2}y_k - \cdots - a_nz^{-n}y_k + b_1z^{-1}u_k + b_2z^{-2}u_k + \cdots b_{nz^{-n}}u_k \tag{9}$$

This prediction form can be written as:

$$y_k = R^TP \tag{10}$$

Where:

$$R = [-y_{k-1} \; - y_{k-2} - \cdots - y_{k-n} + u_{k-1} + -u_{k-2} - u_{k-n}]^T \tag{11}$$

$$P = [a_1a_2 \ldots \ldots \ldots \; a_nb_1b_2 \; \ldots \ldots \ldots \; b_n]^T \tag{12}$$

R is the regression vector obtained from input and output measurements of the system, while P is the parameter vector to be obtained. Note that P does not correspond to the DC motor parameters defined in Section II_A but to the parameters of the ARX model structure. The process of obtaining these parameters and substituting them into (10) is called system identification.

$$R = [-y_{k-1} \quad -y_{k-2} u_{k-1} u_{k-2}]^T, \quad P = [a_1 a_2 b_1 b_2]^T \tag{13}$$

A minimum of 4 data points are required to solve the set of simultaneous equations to obtain these 4 parameters in (13). But the richer the data set, the closer the estimated model parameters approach the true model parameters. It is the fact that the model depends on the outputs that are autoregressive with exogenous inputs that gives the model structure the name ARX.

2.0 MATERIALS AND METHODS

In this article, two cases of the DC motor whose model parameters are to be estimated were examined. These are the noise-free DC motor and the noisy DC motor. The noise-free DC motor is an idealized case that is much easier to estimate than the noisy DC motor. This is because the solution of the set of simultaneous equations leading to the obtaining of the model parameters converges to the actual parameter value. This is not true for the noise system, where the parameters are obtained from a range of values.

A. Noise-free DC motor Model Parameter Estimation

Figure 2 shows the noise-free DC motor stem plot for input and output.

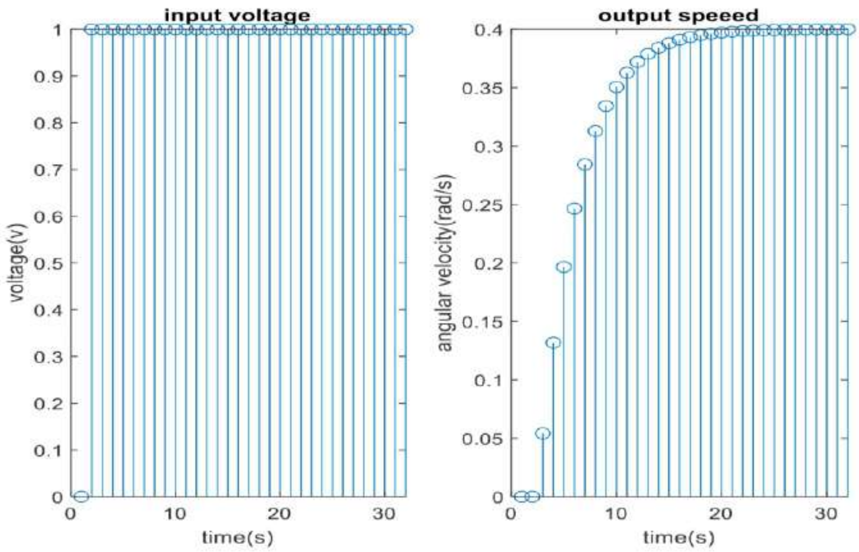

Figure 2: Input and output of a noise-free DC motor

This is obtained from the discrete step response using a simulation time of 3.1 seconds with a sampling time of 0.1s, giving 32 data points. It is observed in Figure 2 that the system has achieved a steady state after 20 samples. Since the DC motor model is a second-order model, there are a minimum of two sample delays; hence, both the measurement and the regression vector have the same number of rows, which is 30. The measured input-output data set for a unit-step voltage input to the DC motor is given in Table 1.

Table 1: Input-output data used for the regression vector

k	u	y
1	0	0
2	1	0
3	1	0.0541
4	1	0.1318
5	1	0.1966
6	1	0.2465
7	1	0.2843
8	1	0.3128
9	1	0.3343
10	1	0.3505
11	1	0.3627
12	1	0.3719
13	1	0.3789
14	1	0.3841
15	1	0.3880
16	1	0.3910
17	1	0.3933
18	1	0.3949
19	1	0.3962
20	1	0.3972
21	1	0.3979
22	1	0.3984
23	1	0.3988
24	1	0.3992
25	1	0.3994
26	1	0.3996
27	1	0.3997
28	1	0.3998
29	1	0.3999
30	1	0.3999
31	1	0.4000
32	1	0.4000

Notice in Table 1 that the DC motor attains a steady state value at the 31st sample, which is about 3 seconds, while from Figure 2, the steady state is seen to have been reached after 20 samples (2 seconds). For this reason, steady states are usually obtained when the outputs enter and stay within a certain value of the final value.

The regression vector for this second-order system is a 30 x 4 matrix, where the first vector is obtained from the output measurement in Table 1. This column starts from the second row (where y=0) to the 31st row (where y=0.4000). The second column starts from the first row (where y = 0) to the 30th row (where y = 0.399). The third column is obtained from the input in Table 1. This column starts from the second row (where u = 1) to the 31st row (where u = 1). The fourth column is the same as the third column, despite starting from the first row to the 30th row. The output in (10) is obtained from the third row (where y = 0.0541) to the last row (where y = 0.4000).

After building the regression vector and the measurement vector, the parameter vector is obtained by solving the set of simultaneous equations according to (10). By using the MATLAB; THE MATHWORKS, INC. (2021) matrix division, the parameter vectors are obtained as:

$$P = [0.9199 - 0.1287\ 0.05400\ 0.0276]^T \tag{14}$$

B. Noisy DC motor Model Parameter Estimation

Gaussian noise of 3% of the steady-state value of the DC motor speed is added to the DC motor output. Figure 3 shows the plot of the inputs and outputs.

In Figure 3, the DC motor response is stochastic; hence, there is difficulty in solving the set of equations that provide the optimal parameter estimate. Applying the same procedure as in III_A, a regression vector and a measurement vector are formed. The corresponding estimated parameters for the noisy system is given as:

$$P = [0.8569 - 0.0771\ 0.0347\ 0.0504]^T \tag{15}$$

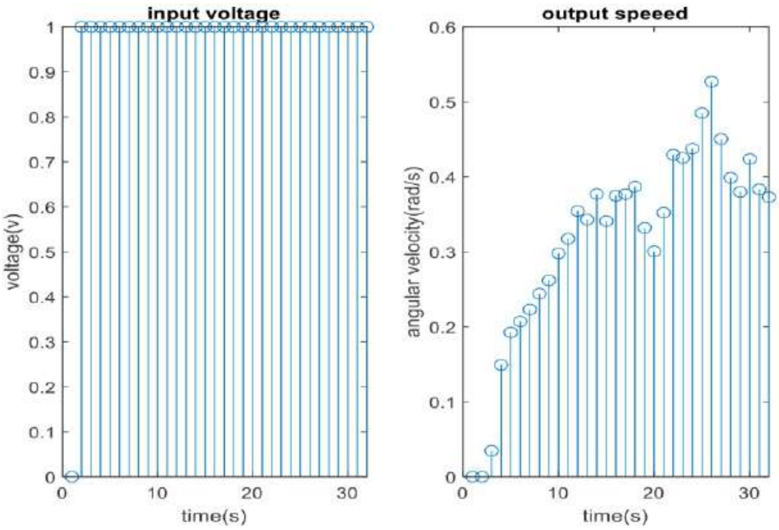

Figure 3. Input and output of a noisy DC motor

However, when the same experiment is repeated 200 times and the parameter estimates are recorded, the optimal parameter estimate is obtained from the mean of the 200 sets of parameter estimates, and it is given as:

$$P1 =]0.8616 - 0.0967\ 0.0476\ 0.0462]^T \tag{16}$$

3.0 RESULTS AND DISCUSSION

In this section, we use the model parameters from solving the set of simultaneous equations to obtain the model of the DC motor. The models are provided in prediction form and in transfer function form for both the noise-free DC motor and the noisy DC motor. The response of the estimated models is then compared.

A. Noise-free DC Motor Model

By substituting (14) for P in (13) and putting (13) into (10), the estimated DC motor model in prediction form becomes:

$$y_k = 0.9199y_{k-1} - 0.1287y_{k-2} + 0.05400u_{k-1} + 0.0276u_{k-2} \tag{17}$$

Putting this into the normal form and taking the Z-transform with all initial conditions zero, the discrete transfer function for the estimated DC motor model becomes:

$$G_{dHat} = \frac{0.05400z + 0.0276}{z^2 - 0.9199z + 0.1287} \tag{18}$$

Figure 4 compares the discrete DC motor model obtained from the discretization of the continuous time model (true) with the estimated discrete DC motor model (estimated) and the continuous time DC motor model. The steady state is the same for the continuous system and the two discrete systems (estimated and true). The true and estimated discrete systems are almost equal, with the slight difference observed after the 0.5-minute mark; hence, the model from the parameter estimate can represent the system accurately in the case of a noise-free system.

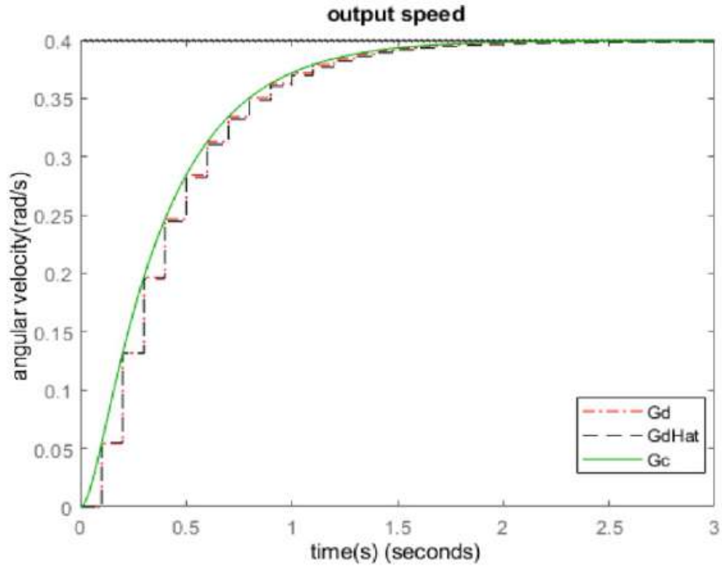

Figure 4:Continuous, true discrete and estimated discrete response of a noise-free DC motor

B. Noisy DC Motor Model

By substituting (15) for P in (13) and putting (13) into (10) similar to section IV_A, the estimated DC motor model in prediction form becomes:

$$y_k = 0.8569y_{k-1} - 0.0771y_{k-2} + 0.0347u_{k-1} + 0.0504u_{k-2} \quad (19)$$

The corresponding transfer function in the discrete time is given as:

$$G_{dHat\_1} = \frac{0.0347z + 0.0504}{z^2 - 0.8569z + 0.0771} \quad (20)$$

Similarly, by substituting (16) for P in (13) and putting (13) into (10), the estimated DC motor model in prediction form becomes:

$$y_k = 0.8616y_{k-1} - 0.0967y_{k-2} + 0.0476u_{k-1} + 0.0462u_{k-2} \quad (21)$$

The corresponding transfer function is:

$$G_{dHat\_2} = \frac{0.0476z + 0.0462}{z^2 - 0.8616z + 0.0967} \quad (22)$$

Figure 5 compares the true discrete DC motor model with two estimated discrete DC motor models. The first estimated model is obtained by performing one experiment, while the other is obtained from the mean of 200 experiments.

In Figure 5, the red dash-dotted graph is the response to the discrete model obtained from the discretization of the continuous time transfer function. The black dashed graph is the response obtained from estimating the DC motor model parameters once, while the green continuous is the corresponding response for the mean of the estimated parameters.

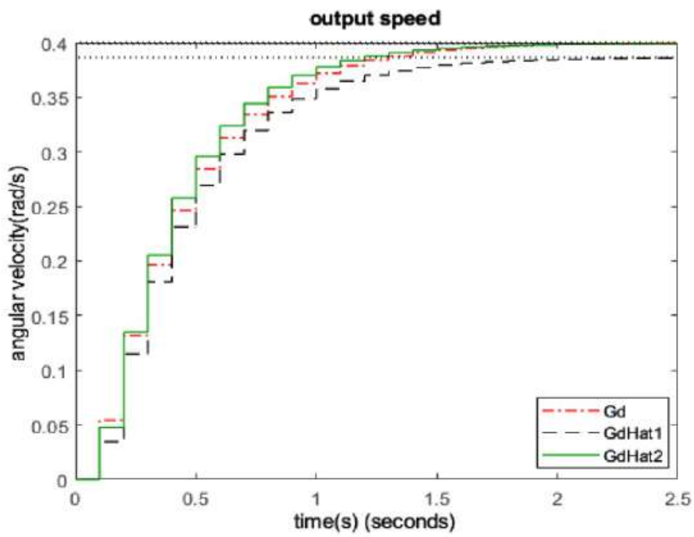

Figure 5:True discrete andestimated discrete response of a noisy DC motor

The steady-state values of the true discrete model response and the mean of the estimated parameters are almost identical. The steady-state value for the response corresponding to estimating the parameter once is different from the response to the true system. This is because the noisy nature of the DC motor speed reduces the accuracy of the estimation process. However, when 200 such experiments were repeated, the corresponding parameter estimates were recorded, and the men were taken, the parameter estimates, hence the model, approached the true system. Hence, for noisy DC motor estimates, taking several input-output readings and finding the mean of the estimated parameters improves the estimation results, but at the expense of computational and memory demands. We also compare the root-mean-square-error (RMSE) of the three estimates above. The RMSE is given as $3.942E\text{-}^{17}$, $3.35E^{-2}$ and $3.15E^{-2}$ for noise-free, noisy, and mean noise. As expected, the noise-free RMSE is negligible (which is highly desirable), while that of the mean estimates is lower (better) than performing the experiment once. Consequently, for noisy DC motor model parameter estimation, a trade-off will have to be made between the number of experiments and the computational and memory demands.

4.0 CONCLUSION

The DC motor ARX model parameter estimation for a noise-free system is straightforward and produces the smallest (and negligible) RMSE. For noisy DC motor, the ARX model parameter estimation is improved when several experiments are performed to obtain corresponding model parameters the mean of which is the optimal parameter estimate. The RMSE when several experiments are performed is smaller than when the estimation is done once. This estimation makes it easier for online model updates for DC motors to take care of parameter variation in the model. This work is limited to the linear DC motor without disturbance. The work can be extended to a nonlinear DC motor and a system with disturbances like varying load torque.

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